A NOTE ON *AN OPEN EXPANDING ECONOMY MODEL*

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In volume 16 (December 1969) of the *Naval Research Logistics Quarterly*, we published a paper, "An Open Expanding Economy Model," in which the well-known von Neumann model, which up to then had been studied by many authors as a closed system, was transformed into the model of an open economy embedded in a "world economy." The consequences of this generalization were sufficiently important to cause immediate response from a number of correspondents. We also presented further work at the Conference on the von Neumann Model arranged by the Institute for Advanced Studies in Vienna, 1970 [5]. In 1971, the original paper [3] appeared with some changes in a French translation [4]. We would here like to record the changes from [3] to [4] for the convenience of readers of this *Quarterly*. We also add remarks on the work by Moeschlin and Rauhut [2] and on our own paper for the Warsaw "Symposium on Mathematical Methods of Economics" of 1972 [6].

After the appearance of [3] Professor J. Łos communicated to us some criticisms by Mrs. L. Mardon concerning the status of Assumption (A6) and its role in subsequent proofs. We acknowledge the correctness of these criticisms (but note that they do not invalidate any of our theorems), and change Assumption (A6) on page 449 of [3] to be the following:

$$\text{(A6)} \qquad t^{-}Ap^{+} > 0.$$

In other words, we change p^{-} in [3] to p^{+} here, as already noted in [4]. Then we change paragraphs 5 and 6 on page 449 of [3] to read as follows:

"The meaning of (A5)* will become clear later when we establish existence proofs. Briefly stated, however, its meaning is the following: *In order to have an economic solution to the open model which is essentially different from a solution to the closed model, then, even when it is operating at its minimum intensities, the economy must produce at least one product having a positive export price.* Stated this way, the requirement is economically significant, since, if this condition were not satisfied, the economy would either not export anything or else export only free goods. And if this were the case, the balance of payments condition would further require that only free goods be imported. Hence if the condition did not hold, the open economy would be, for all practical purposes, closed anyhow.

"Similarly the meaning of (A6) is the following: *when the economy evaluates its goods at the lowest possible prices, the export prices* p^{+}, *then even when operating at its minimum intensities the economy must have a positive demand to input at least one good which has a positive export price.* What this assumption prevents is the possibility of operating at high intensity an industry that requires only inputs that are free, because they have a zero export price, and producing goods that have positive value. Stated in this way the condition is economically meaningful."

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*The Axiom (A5) was $t^{-}Bp^{+} > 0$.

Because of these changes, further changes are required in the first paragraph on page 453 of [3]. The corrected paragraph should read as follows:

"Next consider the optimal solution when α is very large. Equation (12) becomes

$$(B-\alpha A)y-z^{+}+z^{-}=0. \tag{29}$$

Since $0 \leq p^{+} \leq y \leq p^{-}$, we have

$$-z^{+}+z^{-}=-(B-\alpha A)y \geq -Bp^{-}+\alpha Ap^{+}. \tag{30}$$

Multiplying on the left by x yields

$$-xz^{+}+xz^{-} \geq -xBp^{-}+\alpha xAp^{+}. \tag{31}$$

Since $t^{-} \leq x \leq t^{+}$, we can set $x=t^{-}$ and deduce the inequality

$$-t^{-}z^{+}+t^{-}z^{-} \geq -t^{-}Bp^{-}+\alpha(t^{-}Ap^{+}). \tag{32}$$

Now assumption (A6) says $t^{-}Ap^{+} > 0$ so that we can make this quantity arbitrarily large by making α large. The objective function of the dual linear programs is $-t^{+}z^{+}+t^{-}z^{-}$ which differs by the term $-(t^{+}-t^{-})z^{+}$ from (32). Because the lefthand side of (29) decreases with α, z^{+} cannot increase with α, and we see that the objective function of the dual programs must also become arbitrarily large as α becomes large. This completes the proof."

The statement of Theorem 1 is unaffected by these changes and Theorem 2 was independent of (A6) at any rate. This raises the question of under what still more general assumptions our results are correct.

We repeat the remark given in [5] that R. L. Weil has provided in [7] a proof of the fact that the value of the linear programming problem discussed in the first paragraph of the proof of Lemma 2 on page 452 of [3] is a continuous function of the expansion parameter α.

In the first footnote on page 448 of [3] we noted that in any *basic* solution to the linear programming problem under consideration the conditions $w_i^{+}w_i^{-}=0$ and $z_j^{+}z_j^{-}=0$ always hold. In [2] Moeschlin and Rauhut properly remark (and in the spirit of our footnote) that nonbasic solutions *not* satisfying these properties may also exist, and that consequently, these conditions may if desired be added as additional assumptions to the model.

In a very interesting paper [1] A. Moeschlin has applied the model to show that by changing the control variables t^{+} and t^{-} it is possible to impose bounds on imports of goods, and that by changing import prices p^{-} it is possible to impose bounds on losses of industries.

Finally, several readers of [3] have asked how the control variables t^{+} and t^{-} (upper and lower bounds for production intensities) are to be fixed by the economy. This interesting question and its relation to long term planning is discussed in detail in our paper presented at the Warsaw symposium [6].

In view of the fact that our various studies of expanding economies extend over many areas and were published in a variety of journals, some of them of difficult access, we are currently engaged in writing a book which will cover in a unified way our previous work and in many cases make substantial extensions.

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[7] Weil, R. L., "Game Theory and Eigensystems," SIAM Review *X*, pp. 360–367 (1968).